Exercise 67

Find the exact value of each expression.

(a) $\cot^{-1}\left(-\sqrt{3}\right)$ (b) $\sec^{-1} 2$

Solution

Inverse trigonometric functions yield angles, specifically angles measured counterclockwise from the positive x-axis to a certain point on the unit circle.

Part (a)

Rewrite the expression in terms of something more familiar.

$$\theta = \cot^{-1}\left(-\sqrt{3}\right)$$

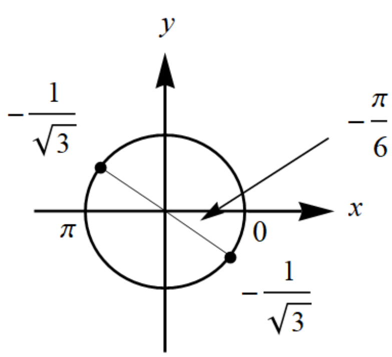
$$\cot\theta = -\sqrt{3}$$

Invert both sides.

$$\frac{1}{\cot\theta} = -\frac{1}{\sqrt{3}}$$

$$\tan\theta = -\frac{1}{\sqrt{3}}$$

There are two points on the unit circle where the tangent is $-1/\sqrt{3}$ as shown below.



The inverse cotangent only gives a value between 0 and π, so the desired point is in the second quadrant.

$$\theta = \cot^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$$

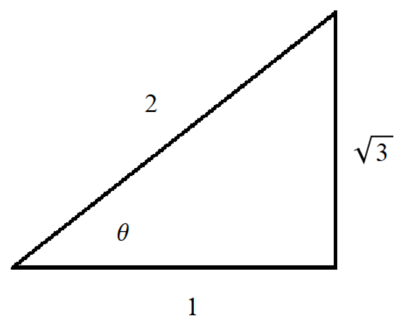
Part (b)

Rewrite the expression in terms of something more familiar.

$$\theta = \sec^{-1} 2$$

$$\sec\theta = 2$$

Draw the implied right triangle, using the Pythagorean theorem to determine the unknown length.



We can see that

$$\sin\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}.$$

Therefore,

$$\sec^{-1} 2 = \frac{\pi}{3}.$$